??? UNKNOWNS IN MULTIPLE SCATTERING

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Summary

Multiple scattering of waves causes attenuation and anisotropic seismic velocities. In addition, if scattering is strong enough, energy propagation becomes diffusive. To study these effects, we developed a medium in which the scattering properties can be easily controlled and the wave motion can be measured *inside* the scattering medium, using an optical detection system.

Introduction

Multiple scattered waves are often regarded as a source of "noise", best removed from the data. Partly this is because many data processing algorithms are based on single-scattering theory, such as the Born approximation; but also, there is a sense that multiple scattered waves are "chaotic" or unstable. Recent work has shown that this need not be the case [Scales and Snieder, 1999] and that multiple-scattered waves can be exploited to make inferences about the scattering medium.

Multiple scattering causes subtle long-wavelength effects such as anisotropy (if the scatterers are aligned) and attenuation (as energy is shifted from the ballistic pulse into the multiple-scattering coda). This is well known in seismology and has been used to interpret effective material properties [Backus, 1962, Aki and Chouet, 1975, O'Doherty and Anstey, 1971] . However, the same wave propagation experiment can be looked at from different points of view (such as ballistic propagation, diffusion or radiative transfer) depending on, for example, the wavelength of the probing beam relative to the size of the disorder and on the distance propagated.

We describe a laboratory model to study multiple scattering of ultrasonic surface waves. An aluminum block has 144 grooves etched in the semi-periodic Fibonacci sequence (Figure 1). An angle-beam transducer launches plane surface waves into this disordered medium and a laser-Doppler vibrometer measures the vertical component of particle velocity on the surface. The grooves are 1 mm wide by 3 mm deep and the dominant wavelength of the surface waves is about 15 mm, so there are many scatterers per wavelength when the waves are propagating perpendicular to the grooves [Scales and Van Wijk, 1999]. Shot gathers contain 10 offsets at a constant angle with respect to the grooves. By varying offset and angle with the grooves, we are able to parametrically vary the strength of the scattering so that we can continuously and reproducibly change the material properties from no scattering to strong multiple-scattering. Examples of a constant offset and angle section are given in Figure 1.

Group velocity and attenuation

The longer effective path lengths of the multiple scattered waves result in a significant slowing down of the energy propagation. To estimate the group velocity as a function of angle, we sorted the data into constant-angle sections and then computed the energy envelopes of each trace. A regression on the peaks of the energy envelopes gives us a the energy propagation speed (left of

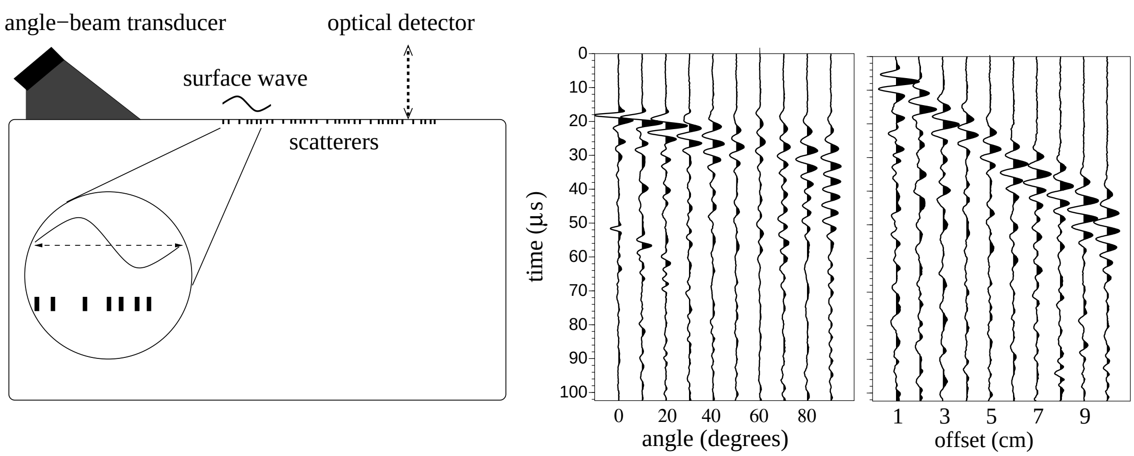

Figure 1: Left: schematic drawing of the experimental set-up. Right: a common offset (5cm) and common angle (50°) gather.

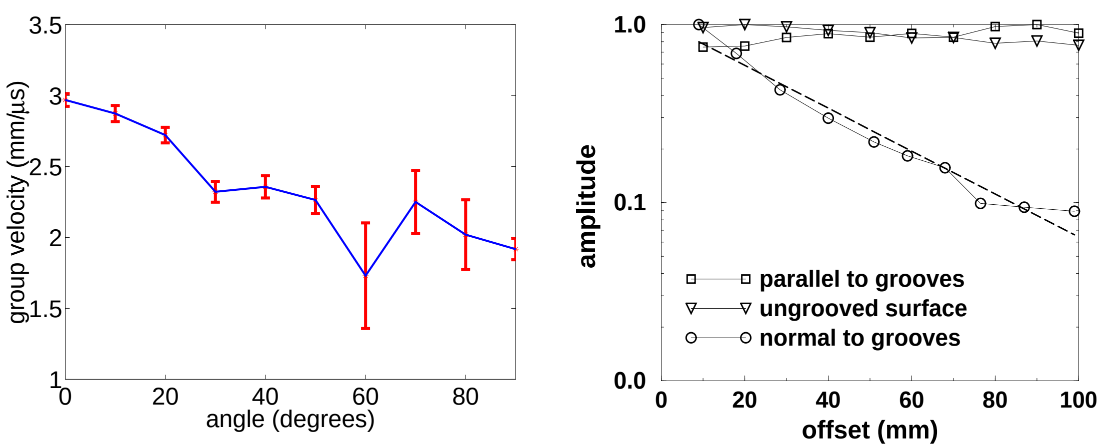

Figure 2: Scattering causes the group velocity to decrease, and phase amplitudes to decay exponentially, because energy is shifted to the coda.

Figure 2). The error bars are 98% coverage intervals from the regression. We see a significant drop in group velocity as a function of angle or scattering strength. On the right side of Figure 2, it is shown that strong multiple scattering causes an exponential decay of signal strength. The amplitudes for waves traveling scattering-free are plotted for reference.

Mean free path

We can exploit the spatial disorder of the Fibonacci grooves [Carpena et al., 1995] to estimate the scattering mean free path of the medium as well as to study the transport of energy. To do this, we need an ensemble of measurements, varying over the disorder in the medium. This allows us to separate multiple scattering effects from inelastic behavior on parameters such as attenuation. Figure 3 shows the ensemble of traces recorded at a fixed 5 cm offset for 38 different positions in the medium.

The total and coherent intensity, or energy, are expected to decay exponentially as [Ishimaru, 1997]: $I_t(x) = I_0 \exp(-x/\ell_a)$ and $I_c(x) = I_0 \exp(-x/\ell_a) \exp(-x/\ell_s)$, where ℓ_a and ℓ_s are the absorption and scattering mean free paths, respectively.

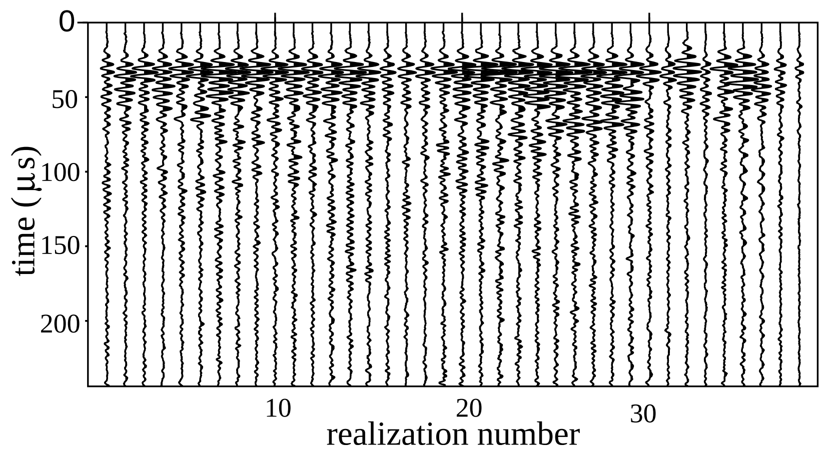

Figure 3: 38 traces recorded at a fixed offset of 5 cm. The source/receiver pair was moved in the sequence of grooves between shots.

If we take the ratio of these two intensities, we get a decay that depends only on the scattering mean free path [Rosny and Roux, 2001]:

$$\frac{I_c(x)}{I_t(x)} = \exp(-x/\ell_s) = \exp(-vt/\ell_s) = \exp(-t/\tau_s), \tag{1}$$

where v is the group velocity and τ_s is the scattering mean free time.

I_c is the intensity of the average trace, while I_t is the average of the intensities of the 38 individual traces. The ratio of I_c to I_t is shown in Figure 4. By fitting an exponential to the portion of the curve after the coherent arrival, we get a mean free time $\tau_s \approx 24\mu$s.

With a transport velocity around 2 mm/μs, the mean free path is just under 5 cm. Thus we are in a regime in which the wavelength is large compared to the size of an individual scatterer, but small compared to the mean free path; while we have measurements with source-receiver offsets as large as 2 mean free paths. In this sense we can see the transition from ballistic to diffusive propagation,

To get some idea of whether the measured wave forms behave diffusively at 5 cm offset, as our estimate of the mean free path suggests, we fit the total intensity with an analytic model associated with propagation in a homogeneous diffusive, absorbing medium. (We included an absorption term in order to account for diffraction losses off the bottom of the grooves.) The Green's function for this model is

$$I(x,t) = (4\pi Dt)^{-1/2} \exp\left(-\frac{x^2}{4Dt} - D\kappa^2 t\right), \tag{2}$$

where $\kappa = 1/\ell_a$ is the absorption coefficient, D is the diffusion constant and x is the propagation distance. The fit is shown in Figure 4. We can get a rough estimate of the diffusion constant as follows. $D = v\ell_{tr}/d$ where v is the transport velocity, ℓ_{tr} is the transport mean free path and d the dimension of the experiment. For $90°$ propagation $v \approx 2\ \mathrm{mm}/\mu$s. The transport mean free path is approximately equal to the scattering mean free path. This would give a diffusion constant of approximately 50 mm^2/μs [Scales and Van Wijk, 2001].

Conclusions

In order to understand the behavior of multiple-scattered waves, we have developed an ultrasonic laboratory model, in which we can vary the strength of the scattering, *and* measure the wave-field

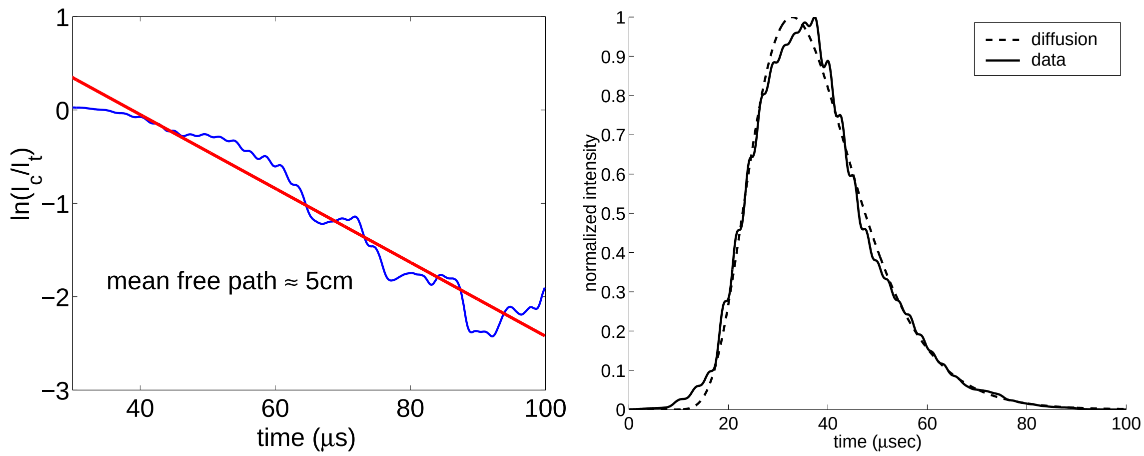

Figure 4: Left: ratio of coherent to total intensities averaged over the ensemble of realizations. This decay is best fit with an exponential with a mean free time of 24 μs. Right: fit of intensity to a 1-D diffusive and attenuating model.

inside the scattering medium.

We observed the effects of multiple scattering on the group velocity, and attenuation and noted a transition from ballistic to diffusive propagation as scattering is increased. This means that model parameters like the diffusion constant and mean free path can play a role in characterization of the medium in the strong scattering regime.

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